

# Eccentric Connectivity Index of Strongly Connected Digraphs

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Topological indices are numerical invariants that represent structural features of graphs and are widely applied in chemical graph theory for predicting physico – chemical and biological properties. The *eccentric connectivity index*, of simple graphs introduced by Sharma et al., in [3], is a distance–cum–degree based descriptor defined as

$$\xi^C(G) = \sum_{u \in V} d_u \text{ecc}(u)$$

where  $d_u$  is the degree of the vertex  $u$  of and  $\text{ecc}(u)$  is the eccentricity of the vertex  $u$  of the graph  $G = (V, E)$ .

This article extends the *eccentric connectivity index* to strongly connected digraphs  $D$  by defining

$$\xi^C(D) = \frac{1}{2} \sum_{u \in V} (d_u^+ + d_u^-) \text{mecc}(u)$$

with respect to the metric, *maximum distance* defined in [1] as  $md(u, v) = \max\{\vec{d}(u, v), \vec{d}(v, u)\}$ . where  $d_u^+$  and  $d_u^-$  are the out-degree and in-degree,  $\text{mecc}(u)$  is the m-eccentricity of the vertex  $u \in V$ . An attempt is also made to find the extremal values for strongly connected digraphs.

**Theorem 1.** *If  $G$  be any connected graph, then the eccentric connectivity index of  $G$  and its symmetric digraph  $\hat{G}$  are same. That is  $\xi^C(G) = \xi^C(\hat{G})$ .*

**Theorem 2.** *If  $D$  is a strongly connected digraph with  $n$  vertices and  $a$  arcs, then  $a.\text{mrad}(D) \leq \xi^C(D) \leq a.\text{mdiam}(D)$ .*

**Corollary 1.** *Let  $D$  be a strongly digraph with  $n$  vertices and  $a$  arcs. Then  $a.\text{mrad}(D) = \xi^C(D) = a.\text{mdiam}(D)$  if and only if  $D$  is self-centered.*

## Eccentric Connectivity Index of Orientations of $K_n$

Different orientations of  $K_n$  may have different eccentric connectivity index. Hence, an attempt is made to find the orientations of  $K_n$ , that have the maximum and minimum eccentric connectivity index.

**Lemma 1.**  $\xi^C(\overleftarrow{K}_n) = \xi^C(K_n) \leq \xi^C(\overrightarrow{K}_n)$ , for any strong orientation of  $K_n$ .

**Theorem 3.** For  $n \geq 3$  there exist an orientation  $\overrightarrow{K}_n$  of the complete graph  $K_n$  such that  $\xi^C(\overrightarrow{K}_n) = \xi^C(K_n) = \xi^C(\overrightarrow{C}_n)$ .

**Theorem 4.** If  $D = (V, A)$  be a strongly connected digraph of order  $n, n \geq 4$  then  $\xi^C(D) \geq 3(n-1)$ , and equality holds if and only if  $D$  is the biorientation of star graph  $\overleftrightarrow{S}_n$ .

## 1 Maximum Eccentric Connectivity Index of Digraphs

Minimum eccentric connectivity of a digraph on  $n$  vertices is attained for the biorientation of the star graph (as in the case of a simple graph) See Thm. 4. Many studies were done to find the upperbounds of eccentric connectivity index of graphs. Thm. 9 of [2] found maximum eccentric connectivity index of graphs on  $n$  vertices.

If  $P_n = u_1 u_2 \dots u_n$  is any (simple) path, then  $P_n^*$  is the digraph obtained from the biorientation of  $P_n$  by adding exactly one of the arcs  $(u_1, u_n)$  or  $(u_n, u_1)$ .

**Theorem 5.** If  $P_n$  is any simple path, then  $\xi^C(P_n) = \xi^C(\overleftrightarrow{P}_n) \leq \xi^C(P_n^*)$ .

The Thm. 5 imply, one can increase the eccentric connectivity index of a digraph by adding arcs without changing the the eccentricities of the vertices. So the problem here is to find the maximum number of arcs that can be added to  $\overleftrightarrow{P}_n$  so that meccentricity remains the same but as number of arcs increases the eccentric connectivity index become maximum.

## References

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